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The Solvent Effect on the Equilibrium of a Chemical Reaction in a Non-electrolyte Solution: Application to the Keto-enol Tautomerism and Dimerization of Nitrogen Dioxide

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A simple lattice theory of a non-electrolyte solution in which the equilibrium of chemical reaction is reached has been presented and compared with the data of the enolization of ethyl acetoacetate in various solvents and with those of the dimerization of nitrogen dioxide in chloroform. Predicted linear relationships has been obtained for the data of the enolization in non-polar solvents, but they could not be obtained for the reaction in ethyl alcohol or for the dimerization of nitrogen oxide in chloroform. The van't Hoff-Dimroth relation has been derived by the present theory.

It is generally found that the equilibrium of a chemical reaction in a liquid solution is influenced by its medium; this is usually known as the solvent effect. The equilibria in electrolyte solutions has been discussed from the point of view of molecular theory, but those in non-electrolyte solutions have usually been treated only in terms of macroscopic thermodynamic quantities, such as activity and activity coefficient[1-3]; they have hardly been discussed at all from the standpoint of molecular theory.

Recently, the idea[4-6] of accounting for the behavior of associated solutions, in which hydrogen bonding and other weak bonding can occur between the molecules, by considering the system as an ideal mixture of the pure component and the associated complexes has been presented. In this case it is assumed that a chemical equilibrium between the monomers and the associated complexes occurs, and that the equilibrium ratio, as represented in terms of their mole fractions, is independent of the concentrations. However, this idea and its assumptions are very doubtful in the case of real mixtures.

Therefore, some further consideration of the non-electrolyte solution in which the equilibrium of the chemical reaction occurs is desirable in order to account for the medium effect. Such a consideration would contribute to a clearer interpretation of the behavior of associated solutions.

The A⇄B Type Reaction in a Solvent.—The system of a non-electrolyte solution in which a simple equilibrium of the chemical reaction, A⇄B, such as keto-enol tautomerism, occurs in an inert solvent, S, will be considered first. Then the lattice model may be applied to this system, the components of which are A, B and S. Although the lattice model does not describe the behavior of real liquid mixtures exactly,[7] it is convenient for calculating any thermodynamic quantity in the case of a multi-component system, and it can be used as one of approximations for real liquid mixtures.

In addition to the usual assumptions[8] of the lattice theory, these further assumptions were made:

1) All molecules are similar in size.

2) There is no long-range molecular interaction.

Then, using a simple zeroth approximation[9] for the sake of simplicity in calculation, one obtains the partition function (PF) of this system as:

$$(\mathrm{PF})=\mathrm{e}^{-N_{\mathrm{B}}\varepsilon_0/\boldsymbol{k}T}f_{\mathrm{A}}{}^{N_{\mathrm{A}}}f_{\mathrm{B}}{}^{N_{\mathrm{B}}}f_{\mathrm{S}}{}^{N_{\mathrm{S}}}\frac{N!}{N_{\mathrm{A}}!\,N_{\mathrm{B}}!\,N_{\mathrm{S}}!}\times\exp\left\{-\frac{z}{\boldsymbol{k}T}\left(\frac{N_{\mathrm{A}}N_{\mathrm{B}}}{N}w_{\mathrm{AB}}+\frac{N_{\mathrm{B}}N_{\mathrm{S}}}{N}w_{\mathrm{BS}}+\frac{N_{\mathrm{S}}N_{\mathrm{A}}}{N}w_{\mathrm{SA}}\right)\right\} \tag{1}$$

where N_{A}, N_{B}, and N_{S} are the numbers of A, B, and C molecules respectively; N is the total number of molecules, $N_{\mathrm{A}}+N_{\mathrm{B}}+N_{\mathrm{S}}$; f_{A}, f_{B}, and f_{S} are the partition functions of A, B, and C

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[1] E. A. Moelwyn-Hughes, "The Kinetics of Reactions in Solution," 2nd Ed., Oxford University Press, London (1950), Chap. VI.

[2] G. N. Lewis and M. Randall, "Thermodynamics," 2nd Ed., Rev. by K. S. Pitzer and L. Brewer, McGraw-Hill, N. Y. (1961), Chap. 20.

[3] J. E. Leffler and E. Grundwald, "Rates and Equilibria of Organic Reactions," John Wiley & Sons, N. Y. (1963).

[4] I. Prigogine and R. Defay, "Chemical Thermodynamics," translated and revised by D. H. Everett, Longmans, London (1954), Chap. XXVI.

[5] I. Prigogine, A. Bellemans and V. Mathot, "The Molecular Theory of Solutions," North-Holland, Amsterdam (1957), Chap. XV.

[6] A. G. Williamson, *Annual Rev. Phys. Chem.*, **15**, 63 (1964).

[7] J. H. Hildebrand and R. L. Scott, "Regular Solution," Prentice-Hall, New Jersey (1962), Chap. 5.

[8] E. A. Guggenheim, "Mixture," Oxford University Press, London (1952), p. 16.

[9] E. A. Guggenheim, ibid., p. 30.

respectively: ε_0 is the energy change in the A$\rightleftarrows$B reaction in the gaseous phase; and w_{AB}, w_{BS}, and w_{SA} are molecular interchange energies between the species identified in the subscript.

The Helmholtz free energy of the system is, then, given by:

$$\frac{F}{\boldsymbol{k}T}=-\ln(\mathrm{PF})=\frac{N_B\varepsilon_0}{\boldsymbol{k}T}-N_A\ln f_A-N_B\ln f_B$$
$$-N_S\ln f_S+N_A\ln\frac{N_A}{N}+N_B\ln\frac{N_B}{N}+N_S\ln\frac{N_S}{N}$$
$$+\frac{z}{\boldsymbol{k}T}\left(\frac{N_AN_B}{N}w_{AB}+\frac{N_BN_S}{N}w_{BS}+\frac{N_SN_A}{N}w_{SA}\right) \tag{2}$$

Differentiating with respect to N_A and N_B, one obtains for the chemical potential μ_A and μ_B;

$$\frac{\mu_A}{\boldsymbol{k}T}=-\ln f_A+\ln x_A+\frac{z}{\boldsymbol{k}T}\{x_B(1-x_A)w_{AB}$$
$$-x_Bx_Sw_{BS}+x_S(1-x_A)w_{SA}\} \tag{3}$$

$$\frac{\mu_B}{\boldsymbol{k}T}=\frac{\varepsilon_0}{\boldsymbol{k}T}-\ln f_B+\ln x_B+\{x_A(1-x_B)w_{AB}$$
$$-x_Sx_Aw_{SA}+x_S(1-x_B)w_{BS}\}\frac{z}{\boldsymbol{k}T} \tag{4}$$

where x_A, x_B, and x_S are the mole fractions of A, B, and S respectively.

$$x_A + x_B + x_S = 1 \tag{5}$$

As in the equilibrium,

$$\mu_A = \mu_B \tag{6}$$

the following expressions can be obtained:

$$\ln K=\ln\frac{x_B}{x_A}=-\frac{\varepsilon_0}{\boldsymbol{k}T}+\ln\frac{f_B}{f_A}$$
$$+\frac{z}{\boldsymbol{k}T}\{(x_B-x_A)w_{AB}+x_S(w_{SA}-w_{BS})\} \tag{7}$$

$$K=\frac{x_B}{x_A}=e^{-\varepsilon_0/\boldsymbol{k}T}\frac{f_B}{f_A}\exp\left[\frac{z}{\boldsymbol{k}T}\{(x_B-x_A)w_{AB}\right.$$
$$\left.+x_S(w_{SA}-w_{SB})\}\right] \tag{8}$$

From Eqs. 7 and 8, it may easily be seen that the equilibrium ratio, $K=x_B/x_A$, where x_A and x_B are the mole fractions of A and B, contains a factor which represents the solvent effect in terms of x_S, w_{SA}, w_{SB}. A similar formula was also obtained by Moelwyn-Hughes,(10) but he did not develop his formula further or apply it to any experimental data.

For the convenience of analyzing the experimental data, formula 7 is transformed as follows. By substituting $x_S=1-x_A-x_B$ into 7, one obtains:

$$\ln K=-\frac{\varepsilon_0}{\boldsymbol{k}T}+\ln\frac{f_B}{f_A}+\frac{z}{\boldsymbol{k}T}(w_{SA}-w_{SB})$$
$$+\frac{z}{\boldsymbol{k}T}(w_{AB}-w_{SA}+w_{SB})x_B$$
$$+\frac{z}{\boldsymbol{k}T}(w_{SB}-w_{SA}-w_{AB})x_A \tag{9}$$
$$= \gamma + \beta x_B + \alpha x_A \tag{10}$$

with:

$$\alpha=\frac{z}{\boldsymbol{k}T}(w_{SB}-w_{SA}-w_{AB})$$
$$\beta=\frac{z}{\boldsymbol{k}T}(w_{AB}+w_{BS}-w_{SA})$$
$$\gamma=\frac{\varepsilon_0}{\boldsymbol{k}T}+\ln\frac{f_B}{f_A}+\frac{z}{\boldsymbol{k}T}(w_{SA}-w_{SB})$$

For $x_S=0$, formula 10 becomes:

$$\ln K^0 = \gamma + \beta x_B{}^0 + \alpha x_A{}^0 \tag{11}$$

where K^0, $x_A{}^0$, and $x_B{}^0$ are the values of K, x_A, and x_B respectively in the absence of a solvent. When 11 is subtracted from 10, the following equation is obtained:

$$\frac{\ln K-\ln K^0}{x_A-x_A{}^0}=\alpha+\beta\frac{x_B-x_B{}^0}{x_A-x_A{}^0} \tag{12}$$

The 2A$\rightleftarrows$B Type Reaction in a Solvent.—This is also a non-electrolyte solution system containing three components, A, B, and S.

In equilibrium,

$$2\mu_A = \mu_B \tag{13}$$

Then, using a procedure similar to that previously used, the following formula can be obtained:

$$\ln K=\ln\frac{x_B}{x_A{}^2}=-\frac{\varepsilon^0}{\boldsymbol{k}T}+\ln\frac{f_B}{f_A{}^2}+\{2x_B-x_Ax_B$$
$$-x_A)w_{AB}+x_S(2-x_A)w_{SA}$$
$$-x_S(1+x_B)w_{SB}\}\frac{z}{\boldsymbol{k}T} \tag{14}$$

This formula will be tested later on the dimerization reaction of nitrogen dioxide.

A Comparison with the Experimental Data of the Keto-enol Isomerization of Ethyl Acetoacetate.—Any adequate comparison of the present formula with the experimental data requires the experimental determination of the mole fractions of the reacting species in non-electrolyte solutions over a wide range of composition. Unfortunately, there are only a few reactions for which such measurements have been made.

As an example of older measurements, the keto-enol isomerizations of ethyl acetoacetate in *n*-hexane, carbon disulfide, benzene and ethyl alcohol were studied by Meyer and Kappelmeier(11) over the whole concentration range. Figure 1

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10) E. A. Moelwyn-Hughes, "Physical Chemistry," 2nd Ed., Pergamon, Oxford (1961), p. 1031.

11) K. H. Meyer and P. Kappelmeier, *Ber.*, **44**, 2718 (1911).

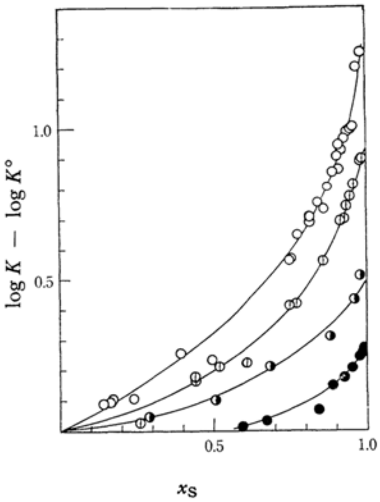

Fig. 1. Variation of equilibrium ratio K for keto-enol isomerization of ethyl acetoacetate with the concentration of the solvent *n*-hexane ○, carbon disulfide ⦶, benzene ◑ and ethyl alcohol ● in their mole fractions. Recalculated from Meyer et al.'s[11] data.

shows the variations in the equilibrium ratio with the solvent concentration, which was recalculated from their data. These recalculated equilibrium ratios, $K=x_B/x_A$, where A and B represent keto and enol forms respectively, were used to test formula 12; the results are shown in Fig. 2. In these tests, the equilibrium ratio, K^0, for a pure reactant was chosen as the standard of comparison because the K^0 ratio was measured most accurately in their measurements. Despite scattering in the original data and a wide variation in the solvent concentration, the plots describe a fairly satisfactory straight line, with a reasonable slope for each solvent except for ethyl alcohol. Alcohol is known as a polar-associated liquid, and the usual simple lattice theory cannot describe the equilibrium behavior of solution; therefore, a straight line perpendicular to the abscissa, which is not in accordance with the prediction of formula 12, might be obtained.

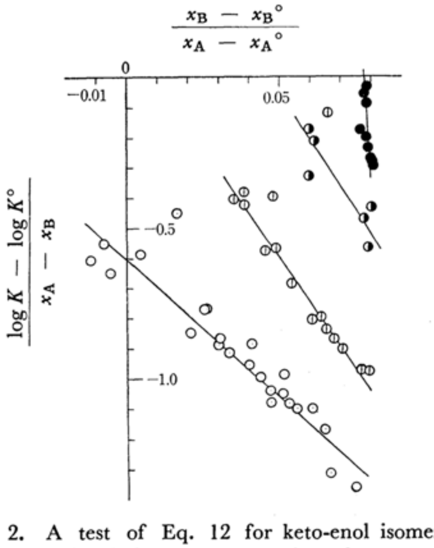

Fig. 2. A test of Eq. 12 for keto-enol isomerization of ethyl acetoacetate in *n*-hexane ○, carbon disulfide ⦶, benzene ◑ and ethyl alcohol ●.

TABLE I. INTERCHANGE ENERGY w_{AB} BETWEEN KETO AND ENOL FORMS OF ETHYL ACETOACETATE IN VARIOUS SOLVENTS

Solvent	w_{AB}, kcal./mol.	
	$z=6$	$z=8$
n-Hexane	−0.9	−0.7
Carbon disulfide	−1.3	−0.9
Benzene	−0.8	−0.6
Ethyl alcohol	—	—

From Fig. 2 the values of α and β can be read. The value of w_{AB}, the molecular interchange energy between A and B, can also be calculated; they are given in Table I for all the solvents. The values of w_{AB} should be independent of the solvent, but the values for the carbon disulfide solvent differ from the values for *n*-hexane and benzene, and the differences exceed the range of experimental error. The difference may be due to the simplicity of the present model and approximation. However, the three values of w_{AB} are in fairly good accordance with each other in spite of the large difference in the influences of the solvents.

The Derivation of the van't Hoff-Dimroth Relation.—For the $A \rightleftarrows B$ reaction, the value of the equilibrium constant, K, given by the $K=C_B/C_A$ equation at a high dilution, varies from solvent to solvent if the concentrations, C_A and C_B respectively, of A and B are expressed either in moles per liter or in any other conventional unit. However, the constant, G (characteristic of the particular reaction), defined by the equation;

$$G = \frac{C_B/S_B}{C_A/S_A} \tag{15}$$

should be more or less independent of the solvent if the solubilities, S_A and S_B respectively, of A and B in the solvent of interest are not too great. This Eq. 15 is called the van't Hoff-Dimroth relation; it was derived by van't Hoff[12] from thermodynamic considerations and was tested by Dimroth[13] for the isomerism of 1-phenyl-5-aminotriazole carboxylic acid esters in various solvents. The relation has been shown to hold also for the equilibrium between tautomeric forms in various solvents in which the solubilities of the reactants are not too great.

It may easily be shown that the relation can be

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[12] J. H. van't Hoff, *Lectures on Theoret. Chem.*, **1**, 221 (1898).
[13] O. Dimroth, *Ann.*, **377**, 127 (1910).

deduced from the present theory. Let us first obtain the solubility, $x_A{}^*$, of A in a solvent, S. In a saturated solution of A in S, we have:

$$\mu_A(\mathrm{s}) = \mu_A(\mathrm{l}) \tag{16}$$

where $\mu_A(\mathrm{s})$ and $\mu_A(\mathrm{l})$ are the chemical potentials of A in its solid and solution states respectively.

By substituting 3 into 16, we obtain:

$$\ln x_A{}^* = \ln\frac{f_A{}^{\mathrm{l}}}{f_A{}^{\mathrm{s}}} - \frac{z}{\boldsymbol{k}T}(1-x_A{}^*)^2 w_{AB} \tag{17}$$

where $f_A{}^{\mathrm{s}}$ and $f_A{}^{\mathrm{l}}$ are the partition functions of A in the solid and liquid states respectively. Similarly, the solubility, $x_B{}^*$, of B in S can be obtained:

$$\ln x_B{}^* = \ln\frac{f_B{}^{\mathrm{l}}}{f_B{}^{\mathrm{s}}} - \frac{z}{\boldsymbol{k}T}(1-x_B{}^*)^2 w_{BS} \tag{18}$$

On the basis of, 17, and 18, the right-hand side of Eq. 15 can be expressed in this form:

$$\begin{aligned}\ln\frac{x_B/x_B{}^*}{x_A/x_A{}^*} &= -\frac{\varepsilon_0}{\boldsymbol{k}T} + \ln\frac{f_A{}^{\mathrm{s}}}{f_B{}^{\mathrm{s}}}\\ &+\frac{z}{\boldsymbol{k}T}\{(w_{AB}+w_{BS}-w_{AS})(x_B-x_A)\\ &+w_{BS}(x_B{}^*-2)x_B{}^*-w_{AS}(x_A{}^*-2)x_A{}^*\}\end{aligned} \tag{19}$$

The conditions under which the van't Hoff-Dimroth relation holds, when the equilibrium is at a high dilution and the solubilities of the reactants are not too great, are:

$$x_A,\ x_B,\ x_A{}^*,\ x_B{}^* \ll 1 \tag{20}$$

Therefore, $x_A{}^{*2}$ and $x_B{}^{*2}$ can be neglected compared with $x_A{}^*$ and $x_B{}^*$, and 19 becomes:

$$\begin{aligned}\ln\frac{x_B/x_B{}^*}{x_A/x_A{}^*} &\simeq -\frac{\varepsilon_0}{\boldsymbol{k}T} + \ln\frac{f_B{}^{\mathrm{s}}}{f_A{}^{\mathrm{s}}}\\ &+\frac{z}{\boldsymbol{k}T}\{(w_{AB}+w_{BS}-w_{AS})(x_B-x_A)\\ &-2(w_{BS}x_B{}^*-w_{AS}x_A{}^*)\}\end{aligned} \tag{21}$$

The third term on the right-hand side of 21, containing solvent parameters, is consisted of differences of small values if the conditions 20 are taken into consideration, and it may become appreciably small compared with the other terms. Therefore, the quantity on the left-hand side of 21 becomes hardly dependent at all on the solvent; it can thus easily be seen that 21 is equivalent to the van't Hoff-Dimroth relation 15.

A Comparison with the Experimental Data of the $2NO_2 \rightleftarrows N_2O_4$ Reaction in Chloroform.—Cundall[14] investigated the dimerization of nitrogen dioxide in chloroform at 0°C over the full range of concentration, and Lewis et al.[15] calculated the equilibrium ratio, $K = x_B/x_A{}^2$, at various values of x_S from Cundall's data; x_A, x_B, and x_S are the mole fractions of nitrogen dioxide, nitrogen tetraoxide and chloroform respectively.

According to their results, the condition,

$$x_A \ll x_B,\ x_S$$

held over nearly the entire range of the concentration. Therefore, formula 14 becomes:

$$\begin{aligned}\ln K \simeq &-\frac{\varepsilon_0}{\boldsymbol{k}T} + \ln\frac{f_B}{f_A{}^2} - \frac{2zw_{AB}}{\boldsymbol{k}T}x_B\\ &+\frac{2z}{\boldsymbol{k}T}(w_{SA}-w_{SB}-w_{AB})x_S + \frac{z}{\boldsymbol{k}T}w_{BS}x_S{}^2\end{aligned} \tag{22}$$

In the absence of a solvent, i. e. when $x_S = 0$, a similar condition,

$$x_A \ll x_B,$$

holds; then 14 becomes:

$$\begin{aligned}\ln K^0 &= -\frac{\varepsilon_0}{\boldsymbol{k}T} + \ln\frac{f_B}{f_A{}^2} - \frac{2zw_{AB}}{\boldsymbol{k}T}x_B\\ &\simeq -\frac{\varepsilon_0}{\boldsymbol{k}T} + \ln\frac{f_B}{f_A{}^2} - \frac{2zw_{AB}}{\boldsymbol{k}T}\end{aligned} \tag{23}$$

By subtracting 23 from 22, one obtains the following equation:

$$\begin{aligned}\frac{\ln K - \ln K^0}{x_S} &= \frac{2z}{\boldsymbol{k}T}(w_{SA}-w_{AB}-w_{SB})\\ &+\frac{z}{\boldsymbol{k}T}w_{BS}x_S\end{aligned} \tag{24}$$

Again the ratio, K^0, for pure a reactant is chosen as a standard because it can be measured more accurately than at a high dilution.

The $(\ln K - \ln K^0)/x_S$ values obtained from Cundall's data are plotted in Fig. 3 against the x_S values, but the predicted linear plot cannot be obtained. This failure may be due to the simplicity of the present lattice model and the rough approximation, to the assumption that the dimers are the same size as the monomer; the long-range molecular interaction is neglected although a considerable change in dipole moment occurs in the dimerization in the polar solvent.

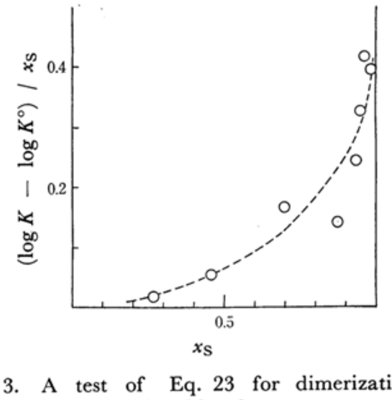

Fig. 3. A test of Eq. 23 for dimerization of nitrogen dioxide in chloroform.

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[14] J. T. Cundall, *J. Chem. Soc.*, **59**, 1076 (1895).

[15] Reference 2, p. 273.

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